Symmetry Origin of Infrared Divergence Cancellation in Topologically Massive Yang-Mills Theory

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Abstract

We manifestly verify that the miraculous cancellation of the infrared divergence of the topologically massive Yang-Mills theory in the Landau gauge is completely determined by a new vector symmetry existing in its large topological mass limit, the Chern-Simons theory. Furthermore, we show that the cancellation theorem proposed by Pisarski and Rao is an inevitable consequence of this new vector symmetry.

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There had been an upsurge in the perturbative investigation of 2+1-dimensional non-Abelian Chern-Simons field theory following a seminal work by Witten [1], in which a miraculous relation between quantum Chern-Simons theory and two-dimensional conformal invariant WZW model was found. Various regularization schemes were utilized in the perturbative calculation [2–8]. Now looking back what remarkable features have been revealed by the perturbative theory, it seems to us that there are three points. In addition to the famous finite renormalization of the gauge coupling, one is an explicit confirmation on an earlier assumption by Jackiw [9] that the Chern-Simons theory is the large topological mass limit of the topologically massive Yang-Mills theory (TMYM) [7,10,11], in which the Chern-Simons term can provide a topological mass to the Yang-Mills field without recourse to the Higgs mechanism [12]; The other is the exposure of a new vector symmetry existing only in the Landau gauge fixed Chern-Simons theory [13], which similar to the BRST symmetry puts the gauge and ghost fields in a nonlinear multiplet and is thus entitled the Landau vector supersymmetry. It was verified that this new symmetry has non-trivial dynamical effects: it protects the Chern-Simons theory from getting quantum correction [14,15]. In particular,

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it is intimately related to the ambiguity of the finite renormalization of the gauge coupling constant: if the regularization method adopted in a concrete perturbative calculation preserves the BRST symmetry but violates the Landau vector supersymmetry, the coupling constant receives a finite renormalization [16], while reversely the coupling constant will keep its classical value [17]. There exists no regularization schemes that can preserve both of the Landau vector supersymmetry and the BRST symmetry simultaneously [16,18].

It was well known that in the Landau gauge TMYM is ultraviolet finite [12,19,20] and its beta function vanishes identically. Consequently, there is no dynamical generated mass scale and the form factor of a Green function can only be the form of $f(p^2/m^2)$ with p being certain external momentum[1]. Thus the fact that the Chern-Simons theory is the large topological mass limit of TMYM means that it can be defined as the infrared limit of TMYM. An equivalent statement is that TMYM is infrared finite. On the other side, the Landau vector supersymmetry arises in the Landau gauge-fixed Chern-Simons theory. In view of these two aspects, we cannot help thinking that the Landau vector supersymmetry, which arises in the low-energy limit of TMYM, probably has some connection with the cancellation of the infrared divergence. The aim of this letter is to give an explicit verification.

The cancellation of the infrared divergence in TMYM was investigated in detail in a beautiful paper by Pisarski and Rao [20]. They first observed that in the one-loop vacuum polarization tensor, the infrared divergences come from the gauge parameter dependent part of the gluon propagator, the ghost loop amplitude, and the combination of the parity-odd pieces from each three-gluon vertex and the gluon propagator in the gluon loop contribution. However, under the choice of the Landau gauge, the gauge dependent part vanishes, the combination of parity-odd pieces in the gluon loop amplitude and the ghost loop contribution unexpectedly cancel in the infrared limit. The same phenomenon happens for the one-loop three-gluon vertex. Based on these observations, Pisarski and Rao concluded that the Chern-Simons part is completely responsible for the cancellation of the infrared divergences in TMYM, despite that their ultraviolet behaviour is greatly different. More concretely, the cancellation of the infrared divergence is determined by the odd parity property of the gauge-fixed Chern-Simons theory, and only in the Landau gauge the Chern-Simons theory can present this feature. This conclusion render them to formulate a cancellation theorem to explain the miraculous cancellation of the infrared divergence in TMYM [20]: In Landau gauge the Chern-Simons theory has only a finite renormalization and its quantum effective action takes the same form as the classical one. The cancellation of infrared divergences contributed by the ghost field and the parity-odd parts of the gauge field is entirely dominated by this theorem.

Despite that this theorem gets the essence of the infrared divergence cancellation, but it looks far from being regarded as the origin of a dynamical phenomenon. Especially, the cancellation takes place at the level of the Green function, this theorem is not clear enough to show how the cancellation occurs among the Green functions. There must be certain symmetry hidden behind this empirical theorem, which will relate various Green functions through the Ward identities, to implement the cancellation of the infrared divergence. Thus

[1]The mass dimension of the coupling constant can be removed by a re-definition of the fields, so the only massive parameter left is the topological mass [7].

we place the hope on the Landau vector supersymmetry to play such a role.

The classical action of TMYM in Euclidean space-time is [12]

$$S = -im \int d^3x \epsilon_{\mu\nu\rho} \left(\frac{1}{2} A_\mu^a \partial_\nu A_\rho^a + \frac{1}{3!} g f^{abc} A_\mu^a A_\nu^b A_\rho^c \right) + \frac{1}{4} \int d^3x F_{\mu\nu}^a F_{\mu\nu}^a, \tag{1}$$

where it is assumed that $m > 0$. To remove the mass dimension of the gauge coupling so that it is explicit that the Chern-Simons theory can be regarded as the large topological mass limit of TMYM at classical level, we rescale the field and the coupling, $A \rightarrow m^{-1/2} A$ and $g \rightarrow m^{-1/2} g$. Consequently, the classical action (1) becomes [7]

$$S = -i \int d^3x \epsilon_{\mu\nu\rho} \left(\frac{1}{2} A_\mu^a \partial_\nu A_\rho^a + \frac{1}{3!} g f^{abc} A_\mu^a A_\nu^b A_\rho^c \right) + \frac{1}{4m} \int d^3x F_{\mu\nu}^a F_{\mu\nu}^a. \tag{2}$$

In the Lorentz gauge-fixing condition, the gauge-fixing and the ghost field parts of the total classical effective action are as the following,

$$S_g = \int d^3x \left[-B^a \partial_\mu A_\mu^a - \partial_\mu \bar{c}^a \left(\partial_\mu c^a + g f^{abc} A_\mu^b c^c \right) + \frac{\xi}{2} (B^a)^2 \right]. \tag{3}$$

As in the usual gauge theory, the gauge-fixed action has the well known BRST symmetry [13],

$$\delta A_\mu^a = D_\mu^{ab} c^b, \quad \delta c^a = -\frac{g}{2} f^{abc} c^b c^c, \quad \delta \bar{c}^a = B^a, \quad \delta B^a = 0. \tag{4}$$

However, in the Landau gauge choice, $\xi = 0$, the large topological mass limit of TMYM, the gauge-fixed Chern-Simons theory has a new BRST-like vector symmetry [13–16],

$$V_\mu A_\nu^a = i \epsilon_{\mu\nu\rho} \partial_\rho c^a, \quad V_\mu B^a = -(D_\mu c)^a, \quad V_\mu c^a = 0, \quad V_\mu \bar{c}^a = A_\mu^a. \tag{5}$$

At quantum level, writing out formally the generating functional with the inclusion of the external sources for the fields and their variations under the supersymmetry (5), one can derive the Ward identities corresponding to the Landau vector supersymmetry of the Chern-Simons theory in a standard way [14,15]. In the topologically massive Yang-Mills theory this symmetry is explicitly broken by the Yang-Mills term, but the broken Ward identity can still be written out [16,17]. Here we take a shortcut to derive the needed identities [16]. Consider a general functional $F[\Phi]$ of the field $\Phi = \left(A_\mu^a, B^a, c^a, \bar{c}^a \right)$, the invariance of the quantum observable

$$\langle F[\Phi] \rangle = \int \mathcal{D}\Phi F[\Phi] e^{-S[\Phi]} \tag{6}$$

under an arbitrary infinitesimal transformation $\Phi \rightarrow \Phi + \delta\Phi$ yields the following identity,

$$\left\langle \frac{\partial F[\Phi]}{\partial \Phi} \delta\Phi - \delta S[\Phi] F[\Phi] \right\rangle = 0. \tag{7}$$

Choosing the functions $F[\Phi] = A_\mu^a(x) \bar{c}^b(y)$ and $F[\Phi] = A_\mu^a(x) A_\nu^b(y) \bar{c}^c(z)$ in TMYM, respectively, the Landau vector supersymmetry transformation (5) leads to

$$\left\langle A^a_\mu(x)A^b_\nu(y)\right\rangle = i\epsilon_{\mu\nu\rho}\partial^x_\rho\left\langle c^a(x)\bar{c}^b(y)\right\rangle + \left\langle A^a_\mu(x)\bar{c}^b(y)V_\nu S_{YM}\right\rangle, \tag{8}$$

and

$$\begin{aligned}\left\langle A^a_\mu(x)A^b_\nu(y)A^c_\rho(z)\right\rangle &= i\epsilon_{\mu\rho\lambda}\partial^x_\lambda\left\langle c^a(x)A^b_\nu(y)\bar{c}^c(z)\right\rangle + i\epsilon_{\nu\rho\lambda}\partial^y_\lambda\left\langle A^a_\mu(x)c^b(y)\bar{c}^c(z)\right\rangle\\ &\quad+\left\langle A^a_\mu(x)A^b_\nu(y)\bar{c}^c(z)V_\rho S_{YM}\right\rangle.\end{aligned} \tag{9}$$

A straightforward calculation gives [16]

$$V_\mu S_{YM} = \mathcal{O}^{(0)}_\mu + \mathcal{O}^{(1)}_\mu + \mathcal{O}^{(2)}_\mu; \tag{10}$$

$$\mathcal{O}^{(0)}_\mu = -\frac{i}{m}\int d^3x\epsilon_{\mu\nu\rho}\partial_\rho c^a\Box A^a_\nu; \tag{11}$$

$$\mathcal{O}^{(1)}_\mu = \frac{i}{m}gf^{abc}\int d^3x\epsilon_{\mu\nu\rho}\partial_\rho c^a\left[-2A^b_\lambda\partial_\lambda A^c_\nu + A^b_\lambda\partial_\nu A^c_\lambda - \left(\partial_\lambda A^b_\lambda\right)A^c_\nu\right]; \tag{12}$$

$$\mathcal{O}^{(2)}_\mu = \frac{i}{m}g^2f^{abc}f^{ade}\int d^3x\epsilon_{\mu\nu\rho}A^b_\lambda\partial_\rho c^c A^d_\lambda A^e_\nu. \tag{13}$$

Eq.(8) shows that the broken Landau vector supersymmetry has established a direct relation between the gauge field and the ghost field propagators. In momentum space, it reads

$$D_{\mu\nu}(p) = \epsilon_{\mu\nu\rho}p_\rho S(p) + D_{\mu\rho}(p)\Omega_{\rho\nu}(p)S(p), \tag{14}$$

where $D_{\mu\nu}(p)$ and $S(p)$ are the propagators of gauge and ghost fields in momentum space, respectively; the composite vertex $\Omega(p)$ at tree level comes from $\mathcal{O}^{(0)}_\mu$. Eq.(14) formally leads to

$$\epsilon_{\mu\nu\rho}p_\rho D^{-1}_{\lambda\mu}(p)S(p) = \delta_{\lambda\nu} - \Omega_{\lambda\nu}(p)S(p). \tag{15}$$

Eq.(9) relates the three-point function of the gauge field to the three-point function of the gauge field and the ghost fields. In momentum space it is

$$\begin{aligned}D_{\mu\alpha}(p)D_{\nu\beta}(q)D_{\rho\gamma}(r)\Gamma^{abc}_{\alpha\beta\gamma}(p,q,r) &= -\epsilon_{\mu\rho\lambda}p_\lambda S(p)D_{\nu\beta}(q)S(r)\Gamma_\beta(p,r,q)\\ &\quad-\epsilon_{\nu\rho\lambda}q_\lambda D_{\mu\alpha}(p)S(q)S(r)\Gamma_\alpha(q,r,p)\\ &\quad+D_{\mu\alpha}(p)D_{\nu\beta}(q)S(r)\widetilde{\Gamma}^{abc}_{\alpha\beta\rho}(p,q,r),\\ p+q+r&=0.\end{aligned} \tag{16}$$

where $\Gamma^{abc}_{\mu\nu\sigma}$ and Γ^{abc}_μ are the three-gluon vertex and the ghost-ghost-gluon vertex, respectively; $\widetilde{\Gamma}^{abc}_{\mu\nu\rho}(p,q,r)$ at tree-level comes from the composite operator $\mathcal{O}^{(1)}_\nu$. Extracting the one particle irreducible (1PI) part, we obtain a relation between the three-gluon vertex and the ghost-ghost-gluon vertex,

$$\begin{aligned}D_{\rho\sigma}(r)\Gamma^{abc}_{\mu\nu\sigma}(p,q,r) &= -\epsilon_{\alpha\rho\lambda}p_\lambda D^{-1}_{\mu\alpha}(p)S(p)S(r)\Gamma^{acb}_\nu(p,r,q)\\ &\quad-\epsilon_{\alpha\rho\lambda}q_\lambda D^{-1}_{\nu\alpha}(q)S(q)S(r)\Gamma^{bca}_\nu(q,r,p) + S(r)\widetilde{\Gamma}^{abc}_{\mu\nu\rho}(p,q,r),\\ p+q+r&=0.\end{aligned} \tag{17}$$

The identities (14) and (17) at tree-level can be easily verified with the bare propagators and vertices. The substitution of (15) into (17) further yields

$$D_{\rho\sigma}(r)\Gamma^{abc}_{\mu\nu\sigma}(p,q,r) = \delta_{\mu\rho}S(r)\Gamma^{abc}_{\nu}(p,r,q) - \delta_{\nu\rho}S(r)\Gamma^{abc}_{\mu}(q,r,p)$$
$$-\Omega_{\mu\rho}(p)S(p)S(r)\Gamma^{abc}_{\nu}(p,r,q) + \Omega_{\nu\rho}(q)S(q)S(r)\Gamma^{abc}_{\mu}(q,r,p)$$
$$+S(r)\widetilde{\Gamma}^{abc}_{\mu\nu\rho}(p,q,r). \tag{18}$$

In the following, we shall show explicitly how the identity (18), born out of the Landau vector supersymmetry, enforces the cancellation of infrared divergence. Consider the vacuum polarization tensor contributed from the skeleton diagrams of the ghost and the gluon loops, we have

$$\Pi^{ab}_{\mu\nu}(p) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\Gamma^{acd}_{\mu\lambda\rho}(p,k,-k-p)D_{\lambda\beta}(k)\Gamma^{bdc}_{\nu\alpha\beta}(-p,k+p,-k)D_{\alpha\rho}(k+p)$$
$$-\int \frac{d^3k}{(2\pi)^3}\Gamma^{acd}_{\mu}(p,k,-k-p)S(k)\Gamma^{bdc}_{\nu}(-p,k+p,-k)S(k+p). \tag{19}$$

Despite that Eq.(19) is written as the form of one-loop Feynman diagram amplitude, it can actually be regarded as a vacuum polarization tensor at any higher order since the insertion of the vertices and propagators in these two one-loop skeletons can be chosen to be any order as desired. The first term in the R.H.S. of Eq.(19) is the contribution from the skeleton of gluon loop and the second one from the ghost loop skeleton, these are the only two sources of the infrared divergence [20]. Substituting Eq.(18) into Eq.(19), we obtain

$$\Pi^{ab}_{\mu\nu}(p) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\Big\{S(-k-p)S(-k)\Big[\delta_{\mu\alpha}\Gamma^{acd}_{\lambda}(p,-k-p,k) - \delta_{\lambda\alpha}\Gamma^{acd}_{\mu}(k,-k-p,p)$$
$$-\Omega_{\mu\alpha}(p)S(p)\Gamma^{acd}_{\lambda}(p,-k-p,k) + \Omega_{\lambda\alpha}(k)S(k)\Gamma^{acd}_{\mu}(k,-k-p,p)$$
$$+\widetilde{\Gamma}^{acd}_{\mu\lambda\alpha}(p,k,-k-p)\Big]\Big[\delta_{\nu\lambda}\Gamma^{bdc}_{\alpha}(-p,-k,k+p) - \delta_{\lambda\alpha}\Gamma^{bdc}_{\mu}(k+p,-k,-p)$$
$$-\Omega_{\nu\lambda}(-p)S(-p)\Gamma^{bdc}_{\alpha}(-p,-k,k+p) + \Omega_{\lambda\alpha}(k+p)S(k+p)\Gamma^{bdc}_{\nu}(k+p,-k,-p)$$
$$+\widetilde{\Gamma}^{bdc}_{\mu\alpha\lambda}(-p,k+p,-k)\Big]\Big\}$$
$$-\int \frac{d^3k}{(2\pi)^3}\Gamma^{acd}_{\mu}(p,k,-k-p)S(k)\Gamma^{bdc}_{\nu}(-p,k+p,-k)S(k+p)$$
$$= \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}S(-k)S(-k-p)\Big[\Gamma^{bdc}_{\mu}(-p,-k,k+p)\Gamma^{acd}_{\nu}(p,-k-p,k)$$
$$-\Gamma^{acd}_{\mu}(p,-k-p,k)\Gamma^{bdc}_{\nu}(k+p,-k,-p) - \Gamma^{acd}_{\mu}(k,-k-p,p)\Gamma^{bdc}_{\nu}(-p,-k,k+p)$$
$$+3\Gamma^{acd}_{\mu}(k,-k-p,p)\Gamma^{bdc}_{\nu}(k+p,-k,-p)\Big]$$
$$+\frac{1}{2}\int \frac{d^3k}{(2\pi)^3}S(-k)S(-k-p)\Big\{\Big[\delta_{\mu\alpha}\Gamma^{acd}_{\lambda}(p,-k-p,k) - \delta_{\lambda\alpha}\Gamma^{acd}_{\mu}(k,-k-p,p)\Big]$$
$$\times\Big[\Omega_{\lambda\alpha}(k+p)S(k+p)\Gamma^{bdc}_{\nu}(k+p,-k,-p) - \Omega_{\nu\lambda}(-p)S(-p)\Gamma^{bdc}_{\alpha}(-p,-k,k+p)\Big]$$
$$+\Big[\Omega_{\lambda\alpha}(k)S(k)\Gamma^{acd}_{\mu}(k,-k-p,p) - \Omega_{\mu\alpha}(p)S(p)\Gamma^{acd}_{\lambda}(p,-k-p,k)\Big]$$
$$\times\Big[\delta_{\nu\lambda}\Gamma^{bdc}_{\alpha}(-p,-k,k+p) - \delta_{\alpha\lambda}\Gamma^{bdc}_{\nu}(k+p,-k,-p)\Big]\Big\}$$
$$+\frac{1}{2}\int \frac{d^3k}{(2\pi)^3}S(-k)S(-k-p)\Big\{\Big[\delta_{\mu\alpha}\Gamma^{acd}_{\lambda}(p,-k-p,k) - \delta_{\lambda\alpha}\Gamma^{acd}_{\mu}(k,-k-p,p)\Big]$$
$$\times\widetilde{\Gamma}^{bdc}_{\nu\alpha\lambda}(-p,k+p,-k) + \widetilde{\Gamma}^{acd}_{\mu\lambda\alpha}(p,k,-k-p)\Big[\delta_{\nu\lambda}\Gamma^{bdc}_{\alpha}(-p,-k,k+p)$$

$$
\begin{aligned}
&-\delta_{\lambda\alpha}\Gamma_{\nu}^{bdc}(k+p,-k,-p)\Big]\Big\}\\
&+\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}S(-k)S(-k-p)\Big\{\Big[\Omega_{\lambda\alpha}(k)S(k)\Gamma_{\mu}^{acd}(k,-k-p,p)\\
&-\Omega_{\mu\alpha}(p)S(p)\Gamma_{\lambda}^{acd}(p,-k-p,k)\Big]\widetilde{\Gamma}_{\nu\alpha\lambda}^{bdc}(-p,k+p,-k)\\
&+\widetilde{\Gamma}_{\mu\lambda\alpha}^{acd}(p,k,-k-p)\Big[\Omega_{\lambda\alpha}(k+p)S(-k)\Gamma_{\nu}^{bdc}(k+p,-k,-p)\\
&-\Omega_{\nu\lambda}(-p)S(-p)\Gamma_{\alpha}^{bdc}(-p,-k,k+p)\Big]\Big\}\\
&+\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}S(-k)S(-k-p)\Big\{\Big[\Omega_{\lambda\alpha}(k)S(k)\Gamma_{\mu}^{acd}(k,-k-p,p)\\
&-\Omega_{\mu\alpha}(p)S(p)\Gamma_{\lambda}^{acd}(p,-k-p,k)\Big]\Big[\Omega_{\lambda\alpha}(k+p)S(k+p)\Gamma_{\nu}^{bdc}(k+p,-k,-p)\\
&-\Omega_{\nu\lambda}(-p)S(-p)\Gamma_{\alpha}^{bdc}(-p,-k,k+p)\Big]\Big\}\\
&+\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}S(-k)S(-k-p)\widetilde{\Gamma}_{\mu\lambda\alpha}^{acd}(p,k,-k-p)\widetilde{\Gamma}_{\mu\alpha\lambda}^{bdc}(-p,k+p,-k)\\
&-\int\frac{d^3k}{(2\pi)^3}S(k)S(k+p)\Gamma_{\mu}^{acd}(p,k,-k-p)\Gamma_{\nu}^{bdc}(-p,k+p,-k). \qquad (20)
\end{aligned}
$$

The first term in the expansion of Eq.(20) is only associated with the contribution from the ghost propagators and the ghost-ghost-gluon vertices, and hence it is the term in the gluon loop skeleton that contains the infrared divergence [20]. The global gauge symmetry requires that the ghost-ghost-gluon vertex at any order must be of the form,

$$
\Gamma_{\mu}^{abc}(p,q,r)=f^{abc}\Gamma_{\mu}(p,q,r). \qquad (21)
$$

Moreover, the Ward identities from the BRST symmetry given in Eq.(4) determines the ghost field propagator and the ghost-ghost-gluon vertex are [20,21],

$$
\begin{aligned}
S(p)&=-\frac{1}{p^2\left[1+\Sigma(p^2/m^2)\right]};\\
\Gamma_{\nu}^{abc}(p,q,r)&=p_{\mu}\gamma_{\mu\nu}^{abc}(p,q,r), \qquad p+q+r=0, \qquad (22)
\end{aligned}
$$

where $\gamma_{\mu\nu}^{abc}$ is the composite ghost-gluon vertex function of the following general form,

$$
\begin{aligned}
\gamma_{\mu\nu}^{abc}(p,q,r)=f^{abc}&\left(\epsilon_{\mu\nu\rho}p_{\rho}A_1+\epsilon_{\mu\nu\rho}q_{\rho}A_2+\epsilon_{\mu\lambda\rho}p_{\lambda}q_{\rho}p_{\nu}A_3+\epsilon_{\mu\lambda\rho}p_{\lambda}q_{\rho}q_{\nu}A_4\right.\\
&+\epsilon_{\nu\lambda\rho}p_{\lambda}q_{\rho}p_{\mu}A_5+\epsilon_{\nu\lambda\rho}p_{\lambda}q_{\rho}q_{\mu}A_6+g_{\mu\nu}A_7+p_{\mu}p_{\nu}A_8+p_{\mu}q_{\nu}A_9\\
&\left.+p_{\nu}q_{\mu}A_{10}+q_{\mu}q_{\nu}A_{11}\right),\\
A_i\equiv A_i(p^2&,q^2,r^2,m), \quad i=1,2,\cdots,11; \quad p+q+r=0. \qquad (23)
\end{aligned}
$$

Using Eqs.(21)–(23), we can easily find that up to a constant coefficient term the first term of Eq.(20) cancels with the contribution from the ghost loop skeleton (i.e. the last term) in the low-energy limit,

$$
\begin{aligned}
&\lim_{p^2\longrightarrow 0}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2}C_V\delta^{ab}S(-k)S(-k-p)\left[\Gamma_{\mu}(p,-k-p,k)\Gamma_{\nu}(k+p,-k,-p)\right.\\
&+\Gamma_{\mu}(k,-k-p,p)\Gamma_{\nu}(-p,-k,k+p)+2\Gamma_{\mu}(p,k,-k-p)\Gamma_{\nu}(-p,k+p,-k)\\
&\left.-\Gamma_{\mu}(-p,-k,k+p)\Gamma_{\nu}(p,-k-p,k)-3\Gamma_{\mu}(k,-k-p,p)\Gamma_{\nu}(k+p,-k,-p)\right]=0, \qquad (24)
\end{aligned}
$$

where C_V is the quadratic Casimir operator in the adjoint representation of the gauge group, $f^{acd}f^{bcd} = C_V\delta^{ab}$.

To complete the proof that the infrared divergence indeed cancels in Eq.(20), we need to show that the remained terms are also free from infrared divergence. The remained terms is composed of the ghost field propagator, the ghost-ghost-gluon vertex and the composite vertices relevant to the Landau vector supersymmetric variation of the Yang-Mills term. The identity (14) yields[2]

$$p_\mu \Omega_{\mu\nu}(p) = 0, \tag{25}$$

and hence $\Omega_{\mu\nu}$ should be the following general form,

$$\Omega_{\mu\nu}(p) = m\epsilon_{\mu\nu\rho}p_\rho B_1\left(\frac{p^2}{m^2}\right) + (p^2\delta_{\mu\nu} - p_\mu p_\nu)B_2\left(\frac{p^2}{m^2}\right). \tag{26}$$

As for the composite ghost-gluon-gluon vertex associated with the Landau vector supersymmetry transformation, $\widetilde{\Gamma}_{\mu\nu\rho}(p,q,r)$, the identity (16) leads to a relation between $\widetilde{\Gamma}_{\mu\nu\rho}$ and the ghost-ghost-gluon vertex,

$$r_\rho\widetilde{\Gamma}_{\mu\nu\rho}(p,q,r) = \epsilon_{\alpha\lambda\rho}p_\lambda q_\rho\left[S(p)D^{-1}_{\mu\alpha}(p)\Gamma_\nu(p,r,q) - S(q)D^{-1}_{\nu\alpha}(q)\Gamma_\mu(q,r,p)\right]. \tag{27}$$

Thus the infrared divergence-free of the ghost-ghost-gluon vertex Γ_μ implies that $\widetilde{\Gamma}_{\mu\nu\rho}$ should also has no infrared divergence. The other argument in favour of the infrared divergence-free of $\widetilde{\Gamma}_{\mu\nu\rho}$ is the observation that at tree-level it is relevant to the three-gluon vertex of the three-dimensional Yang-Mills theory,

$$\begin{aligned}\widetilde{\Gamma}^{(0)}_{\mu\nu\rho}(p,q,r) &= -\frac{i}{m}\epsilon_{\nu\lambda\sigma}r_\sigma\left[g_{\mu\lambda}(p-q)_\rho + g_{\lambda\rho}(q-r)_\mu + g_{\rho\mu}(r-p)_\lambda\right]\\ &= -\frac{i}{m}\epsilon_{\nu\lambda\sigma}r_\sigma\Gamma^{(0)}_{(YM)\mu\lambda\rho}(p,q,r),\end{aligned} \tag{28}$$

where $\Gamma^{(0)}_{(YM)\mu\lambda\rho}$ is the tree-level three-gluon vertex of the Yang-Mills part of TMYM. There is no infrared divergence in three dimensional Yang-Mills theory due to its superrenormalizability [22]. The relation (28) will probably be modified at quantum level by quantum correction, but we still think that it gives a favourable support on the infrared divergence-free of $\widetilde{\Gamma}_{\mu\nu\rho}$.

The information collected in Eqs.(22), (23), (26), (27) and (28) on propagators and vertices enable us to use the inductive method to prove the infrared divergence-free of the remained terms of Eq.(20). The procedure is as following. We first show by concrete calculation that in dimensional regularization the terms other than the first and the last ones in Eq.(20) have the $p^2\to 0$ limit, then assume that this is satisfied at the nth order; After a lengthy analysis we find that at $n+1$ order the remained terms of Eq.(20) indeed

[2] In fact, Eq.(14) yields two possibilities, $p_\mu\Omega_{\mu\nu}(p) = 0$ or proportional to p_ν, but the tree level and one-loop results [16], $\Omega^{(0)}_{\mu\nu}(p) = -p^2/m\epsilon_{\mu\nu\rho}p_\rho$ and $\Omega^{(1)}_{\mu\nu}(p) = 3/(4\pi)g^2C_V(p^2\delta_{\mu\nu} - p_\mu p_\nu)$, exclude the second one.

have no infrared divergence. Based on this observation, we conclude that the cancellation of the infrared divergence has occurred in the whole vacuum polarization tensor.

Similar analysis are applied to the quantum three-gluon vertex. It is shown that with Eqs.(18) the infrared divergences coming from the ghost loop skeleton and the pure ϵ-parts of the gluon loop skeleton cancel[3],

$$
\begin{aligned}
&\lim_{p^2,q^2,r^2\to 0}\left[\Gamma^{abc}_{(gl)\mu\nu\rho}(p,q,r)+\Gamma^{abc}_{(gh)\mu\nu\rho}(p,q,r)\right]\\
&=\lim_{p^2,q^2,r^2\to 0}\left\{\int\frac{d^3k}{(2\pi)^3}\Gamma^{aa_1a_2}_{\mu\mu_1\mu_2}(p,-p-k,k)D_{\mu_2\rho_1}(k)\Gamma^{ca_2b_1}_{\rho\rho_1\rho_2}(-p-q,-k,p+q+k)\right.\\
&\quad\times D_{\rho_2\nu_1}(p+q+k)\Gamma^{bb_1a_1}_{\nu\nu_1\nu_2}(q,-p-q-k,p+k)D_{\nu_2\mu_1}(p+k)\\
&\quad-\int\frac{d^3k}{(2\pi)^3}\left[\Gamma^{aa_1a_2}_{\mu}(p,-p-k,k)S(k)\Gamma^{ca_2b_1}_{\rho}(-p-q,-k,p+q+k)S(k+p+q)\right.\\
&\quad\times\Gamma^{bb_1a_1}_{\nu}(q,-k-p-q,k+p)S(k+p)+\Gamma^{aa_1a_2}_{\mu}(p,k+q,-k-p-q)S(k+q)\\
&\quad\left.\left.\times\ \Gamma^{bb_1a_1}_{\nu}(q,k,-k-q)S(k)\Gamma^{ca_2b_1}_{\rho}(-p-q,k+p+q,-k)S(k+p+q)\right]\right\}=0, \qquad (29)
\end{aligned}
$$

the three-gluon vertex function is thus infrared finite. In above equation, $\Gamma^{abc}_{(gl)\mu\nu\rho}$ and $\Gamma^{abc}_{(gh)\mu\nu\rho}$ denote the quantum three-gluon vertex contributed by the gluon loop skeleton and the ghost loop skeleton, respectively.

Having proved that the Landau vector supersymmetry protects the topologically massive Yang-Mills theory from getting the infrared divergence, in the following we shall have a look at the relation between the Landau vector supersymmetry and the cancellation theorem proposed by Pisarski and Rao [20].

First, the infrared cancellation theorem makes it possible for the Landau vector supersymmetry to persist after the quantum correction. The main content of Pisarski and Rao's infrared divergence cancellation theorem is that in the Landau gauge the Chern-Simons term of TMYM keeps its classical form up to a finite wave function and vertex renormalization, i.e. the quantum effective action of Chern-Simons theory is

$$
\begin{aligned}
\Gamma_{CS}[A,c,\bar{c},B] &\stackrel{\xi=0}{=} -i\int d^3x Z_A\epsilon_{\mu\nu\rho}\left(\frac{1}{2}A^a_\mu\partial_\nu A^a_\rho+\frac{1}{3!}\frac{Z_V}{Z_A}g_Rf^{abc}A^a_\mu A^b_\nu A^c_\rho\right)\\
&\qquad -\int d^3x\left[B^a\partial_\mu A^a_\mu+Z_c\partial_\mu\bar{c}^a\left(\partial_\mu c^a+\frac{Z'_V}{Z_c}g_Rf^{abc}A^b_\mu c^c\right)\right]\\
&= -i\int d^3x Z_A\epsilon_{\mu\nu\rho}\left(\frac{1}{2}A^a_\mu\partial_\nu A^a_\rho+\frac{1}{3!}Z_A^{1/2}gf^{abc}A^a_\mu A^b_\nu A^c_\rho\right)\\
&\qquad -\int d^3x\left[B^a\partial_\mu A^a_\mu+Z_c\partial_\mu\bar{c}^a\left(\partial_\mu c^a+Z_A^{1/2}gf^{abc}A^b_\mu c^c\right)\right], \qquad (30)
\end{aligned}
$$

where the fields are all renormalized ones; g and g_R are the bare and the renormalized gauge couplings, respectively; Z_A and Z_g are the wave function renormalization constants for the

---

[3]In concrete perturbative calculation, depending on the choice of a regularization scheme, Eq.(29) at one-loop may have a finite term, $C_1\epsilon_{\mu\nu\rho}+C_2/m[\delta_{\mu\nu}(p-q)_\rho+\delta_{\nu\rho}(q-r)_\mu+\delta_{\rho\mu}(r-p)_\mu]$ with C_1 and C_2 being constants.

gauge and ghost fields, respectively; Z_V and Z'_V are the vertex renormalization constants for the three-gluon vertex and the ghost-ghost-gluon field vertex, respectively. Further, the Slavnov-Taylor identity from the quantum BRST symmetry requires that $Z_V/Z_A = Z'_V/Z_c$ [20,21]. The form (30) clearly shows that that the quantum Chern-Simons action has the following renormalized Landau vector supersymmetry,

$$
\begin{aligned}
&V_{R\mu}A^a_\nu = iZ_A^{-1/2}Z_c^{1/2}\epsilon_{\mu\nu\rho}\partial_\rho c^a, \quad V_{R\mu}c^a = 0, \quad V_{R\mu}\bar{c}^a = Z_c^{-1/2}Z_A^{1/2}A^a_\mu, \\
&V_{R\mu}B^a = -Z_A^{1/2}Z_c^{1/2}\partial_\mu c^a - Z'_V Z_A^{1/2}Z_c^{-1/2}g_R f^{abc}A^b_\mu c^c \\
&\qquad\quad = -Z_A^{1/2}Z_c^{1/2}\partial_\mu c^a - Z_V Z_c^{1/2}Z_A^{-1/2}g_R f^{abc}A^b_\mu c^c.
\end{aligned} \tag{31}
$$

Since the Landau vectors supersymmetry transformation changes parity, so any parity-even terms generated from the quantum correction such as $(F_{\mu\nu})^2$ etc will break the Landau vector supersymmetry. Therefore, the above infrared divergence cancellation theorem guarantees that Landau vector supersymmetry is completely anomaly free.

On the other side, there had appeared several different ways proving that once the Landau vector supersymmetry is imposed, the Ward identity corresponding to it will enforce that the quantum Chern-Simons action to be the form of (30) [14,15]. The first method is combining the Landau vector supersymmetry and the BRST transformation invariance together to form a $N = 1$ supersymmetry algebra, then with respect to this superalgebra, arranging the energy-momentum tensor, the ghost number current, the BRST current and a tensor current corresponding to the Landau vector supersymmetry into a supermultiplet [14]. With the fact that the anomalies of the component currents in a supermultiplet also constitute a supermultiplet [23], it was shown by making use of various Ward identities that the quantum Chern-Simons theory is scale invariant, its beta function and the anomalous dimensions of every fields vanish identically. This means that the quantum Chern-Simons term keeps its classical form [23]. The second way is considering the Ward identity implied by the Landau vector supersymmetry and the Schwinger-Dyson equations for the propagators of the ghost and gauge fields [15]. This method elegantly manifested the vanishing of the radiative correction to the Chern-Simons theory [15]. In addition, the concrete perturbative calculation in a regularization scheme preserving the Landau vector supersymmetry also explicitly confirms that the Chern-Simons action indeed receives no quantum correction [2]. In fact, even one chooses a regularization schemes violating the Landau vector supersymmetry in the perturbative Chern-Simons theory, one typical example of which is the hybrid regularization of higher covariant derivative plus dimensional regularization with the Yang-Mills action as the higher covariant derivative term, the explicit calculation shows that after the regulator is removed the theory still keeps its classical form up to a finite renormalization of the gauge coupling [3–8]. Thus the cancellation theorem is an inevitable consequence of the Landau vector supersymmetry. Based on above arguments considered from both sides, we conclude that the existence of Landau vector supersymmetry of Chern-Simons theory is equivalent to the cancellation cancellation theorem proposed by Pisarski and Rao [20].

In summary, we have verified that the infrared divergence cancellation of topologically massive Yang-Mills theory in the Landau gauge originates completely from a new vector symmetry in the gauge-fixed Chern-Simons term. This is an unusual property of TMYM. It is well known that in four-dimensional case, the infrared divergences arise in a massless field theory. They cancel out in the scattering cross sections when the degeneracy of a

real particle with massless particle states is considered as summarized by the celebrated Kinoshita-Lee-Nauenberg theorem [24], where cancellation has no explicit relevance with the symmetries of the theory. While here in three-dimensional TMYM, the cancellation of the infrared divergence can adhere to certain symmetry of the theory. Thus it is significant to point out this exotic feature.

Furthermore, we have analyzed the relation between the Landau vector supersymmetry and the infrared divergence cancellation theorem proposed earlier by Pisarski and Rao. On one hand, the cancelation theorem ensures that the Landau vector supersymmetry will persist after quantum correction; On the other hand, the Landau vector supersymmetry determine that infrared divergences are doomed to cancel. Consequently, Chern-Simons theory can be defined as the large topological mass limit of TMYM and Pisarski and Rao's cancellation theorem arises. However, there is a difference between the Landau vector supersymmetry and the cancellation theorem: the Landau vector supersymmetry has displayed the origin of the infrared divergence cancellation of TMYM in the Landau gauge, while the cancellation theorem has only emphasized the consequence of the infrared divergence cancellation .

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